Cellpath: state estimation of static traffic networks via convex optimization

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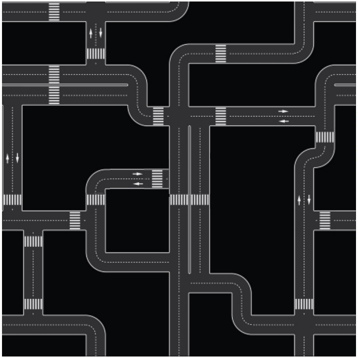
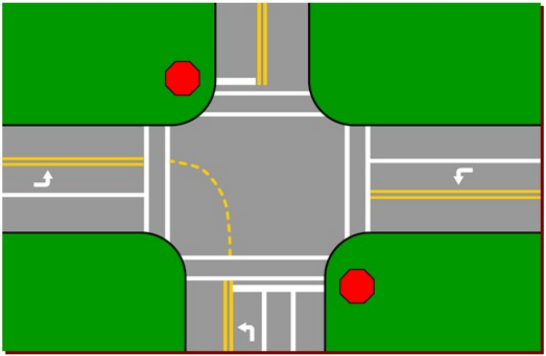
Outline

Route flow estimation problem

The root of all traffic evils

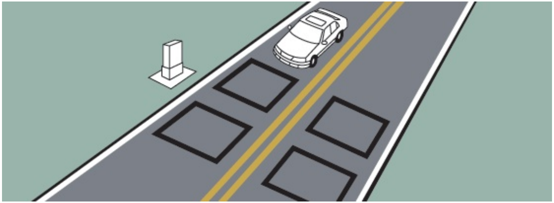
We have little understanding of what's going on in the road network.





Estimation

Traditional approaches to traffic estimation



Sparse static sensors

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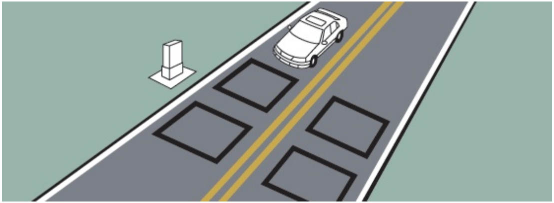
Equilibrium models

This talk: data-driven estimation of route flow



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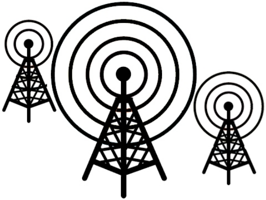
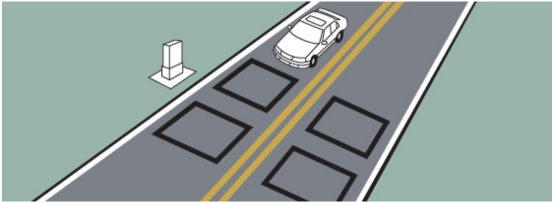


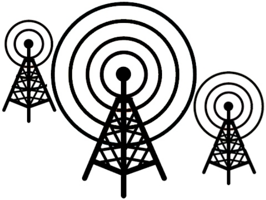
Cellular network sensors

This talk: data-driven estimation of route flow



Sparse static sensors $\rightarrow$ observed link flow

$+$



Cellular network sensors $\rightarrow$ cellpath flow

Problem statement: route flow estimation

Route flow estimation problem

Given

- Road network, origins, cells
- Top routes between origin-destination (OD) pairs
- Cellpath flows, f
- OD flows, d
- Observed link flows, b

Recover

- Flow along routes, x

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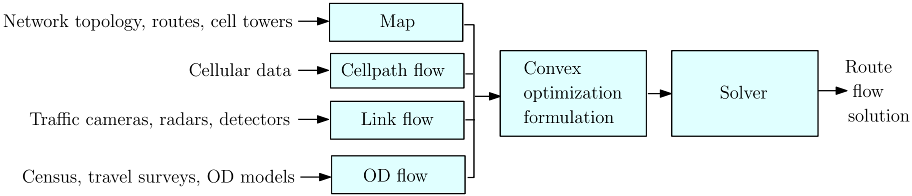
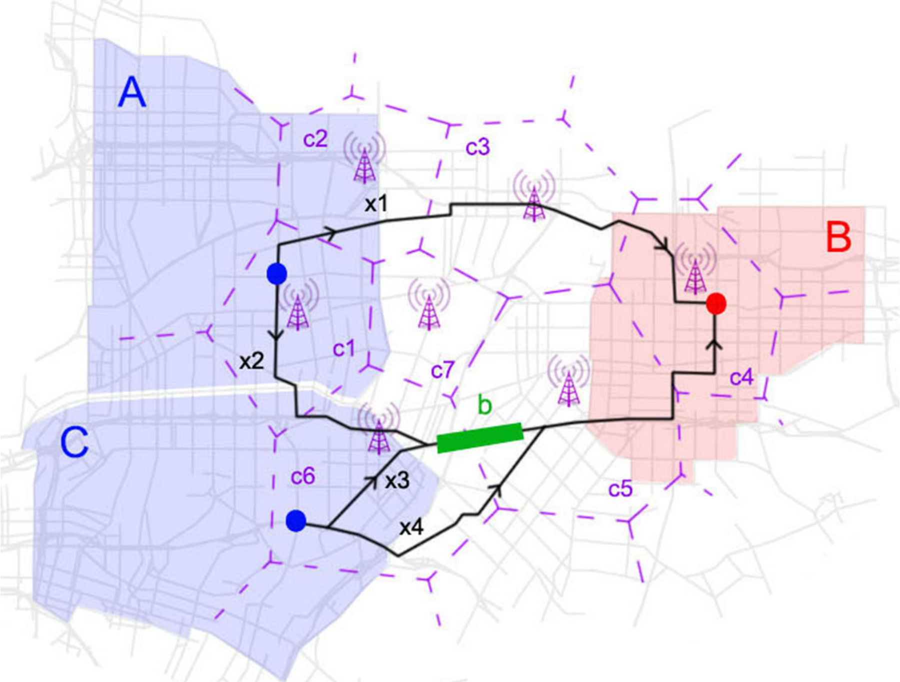


Problem setup



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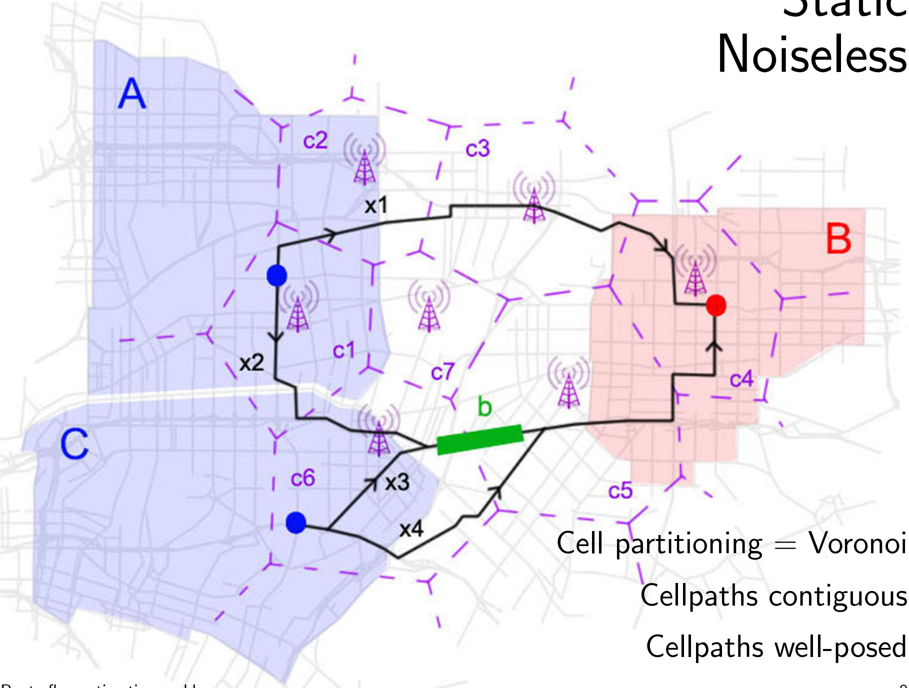


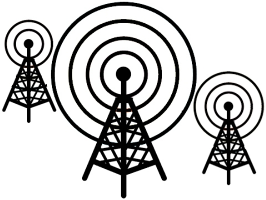
Static
Noiseless

Cell partitioning = Voronoi

Cellpaths contiguous

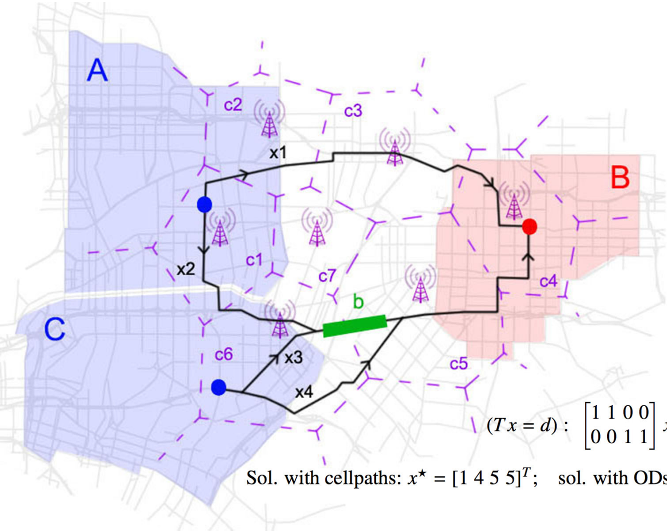
Cellpaths well-posed

Intermission 1: cellular networks crash course



- Not GPS
- Cell towers spaced $\frac{1}{4} - \frac{1}{2}$ miles apart (dense urban areas) to 1-2 miles apart (suburbia)
- Connection to a tower depends on signal strength, current load of nearby towers, hysteresis, etc.
- Cellular signaling data
  - Handovers (HO)
  - Location updates (LU)
  - Call detail records (CDR)

Example problem setup



All flows are in 1000 vehicles/hour.

$$\text{cellpath flows:} \begin{cases} f_{p1234} = 1 \; = x_1 \\ f_{p1654} = 4 \; = x_2 \\ f_{p654} = 10 = x_3 + x_4 \end{cases}$$

$$\text{OD demands:} \begin{cases} d_{AB} = 5 \; = x_1 + x_2 \\ d_{CB} = 10 = x_3 + x_4 \end{cases}$$

$$\text{Link flow: } b = 9 = x_2 + x_3$$

$$(Ux = f): \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} x = \begin{bmatrix} f_{p1234} \\ f_{p1654} \\ f_{p654} \end{bmatrix}$$

$$(Tx = d): \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} x = \begin{bmatrix} d_{AB} \\ d_{CB} \end{bmatrix} \quad ; \quad (Ax = b): \; [0 \; 1 \; 1 \; 0]\, x = b$$

Sol. with cellpaths: $x^\star = [1\;4\;5\;5]^T$; sol. with ODs: $x = x^\star + \begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix}^T t, \; \forall\, t \in [-1, 4]$

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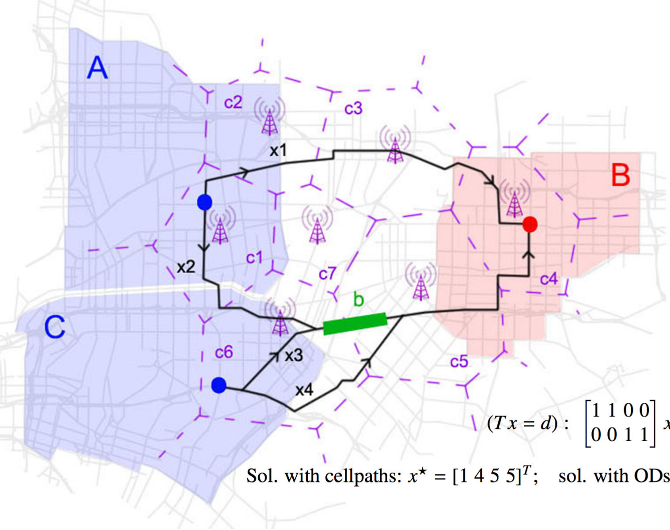

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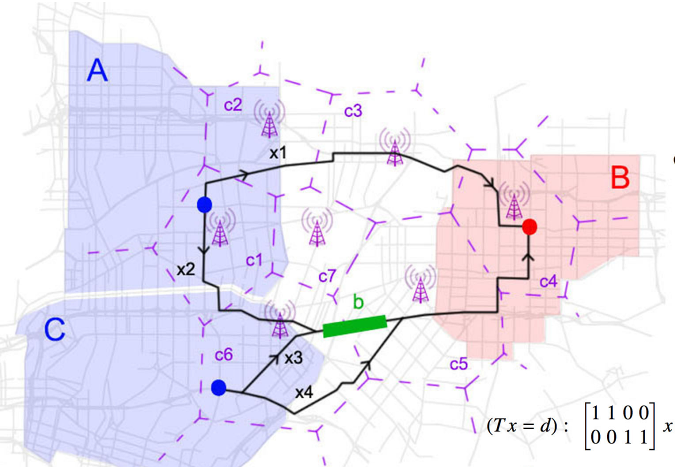

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Intermission 2: crash course on constrained optimization

Constrained quadratic program (QP):

$$\begin{array}{rl} \min & \frac{1}{2}\|Ax - b\|_2^2 \\ \text{s.t.} & Ux = f,\ x \geq 0 \end{array}$$

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5: **end while**
6: **return** x

Technical contributions of our work

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Nice structure of constraints: block simplex
Projection step transformable into ordering constraints:

$$x = x_0 + Nz \geq 0 \quad \Longleftrightarrow \quad 0 \leq z_1 \leq \cdots \leq z_{n-1} \leq 1$$

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$$\begin{array}{rl} \text{minimize} & \sum_{i=1}^{n} w_i(y_i - x_i)^2 \\ \text{subject to} & x_1 \leq x_2 \leq \cdots \leq x_n \end{array}$$

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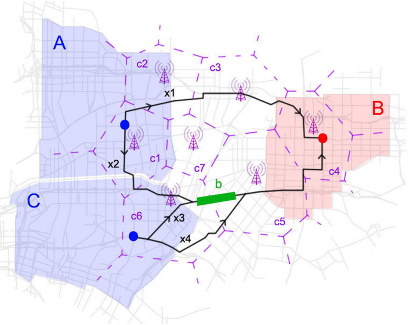
Solvable via Pool adjacent violators (PAV) algorithm solves this in $O(n)$ (1972, 1984)

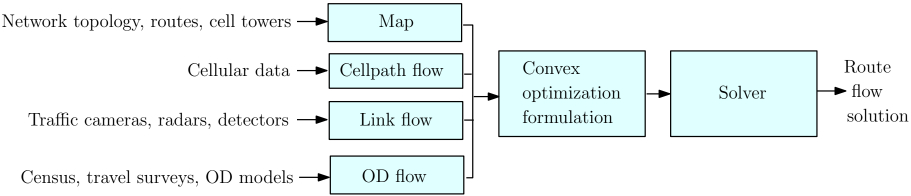
Experiment: Los Angeles network $\implies$ 90% accuracy

- Network: 10,538 nodes, 20,476 links
- 1,033 observed links (5% coverage); 1,000 cells
- 31,836 origin-destination (OD) pairs; 321 ODs
- 295,650 routes (up to 50 routes per OD pair)

Conclusions

- Route flow estimation has received little attention due to data limitations
- Cellular network data is a promising data source
- Route flow estimates will enable short time horizon applications such as prediction and control
- Next up: experiments with AT&T data





- Get in touch: `cathywu@eecs.berkeley.edu`; website: wucathy.com